**MATHEMATICS METHODS**

**MAWA Semester 1 (Unit 3) Examination 2019**

**Calculator-assumed**

# Marking Key

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**The release date for this exam and marking scheme is 14th June.**

**Section Two: Calculator-assumed (100 Marks)**

**Question 8(a) (4 marks)**

|  |
| --- |
| Solution |
| (i) (ii)  =  =  =  =  =  |
| Mathematical behaviours | Marks |
| * states
* uses linearity and additivity to deduce
* anti-differentiates
* determines correct result of 16
 | 1111 |

**Question 8(b) (3 marks)**

|  |
| --- |
| Solution |
| = =  =  =  |
| Mathematical behaviours | Marks |
| * anti-differentiates correctly
* substitutes limits of integration correctly
* determines exact result
 | 111 |

**Question 9(a) (1 mark)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Mark |
| * states  justifies answer using anti-differentiation
 | 1 |

**Question 9(b) (1 mark)**

|  |
| --- |
| Solution |
| For stationary points,    |
| Mathematical behaviours | Marks |
| * states  coordinates of stationary points
 | 1 |

**Question 9(c) (3 marks)**

|  |
| --- |
| Solution |
|  and Hence there is a change in concavity at and  so there is a horizontal point of inflection at . Hence *m* = 1. |
| Mathematical behaviours | Marks |
| * states  and
* demonstrates change in concavity at
* states that horizontal point of inflection occurs at *m* = 1.
 | 111 |

**Question 9(d) (3 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * sketch shows  and roots
* clearly shows  coordinate of minimum turning point
* graphs correct shape and clearly labels points of inflection
 | 111 |

**Question 10(a) (1 mark)**

|  |
| --- |
| Solution |
|  has a binomial distribution with parameters and  ie   |
| Mathematical behaviours | Marks |
| * identifies binomial distribution and states parameters
 | 1 |

**Question 10(b) (1 mark)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * states correct answer
 | 1 |

**Question 10(c) (3 marks)**

|  |
| --- |
| Solution |
|    (from calculator) |
| Mathematical behaviours | Marks |
| * states a probability inequality relevant to one of the  values
* calculates one probability correctly
* calculates all probabilities correctly
 | 111 |

**Question 10(d) (1 mark)**

|  |
| --- |
| Solution |
|  as  |
| Mathematical behaviours | Marks |
| * obtains correct answer
 | 1 |

**Question 10(e) (3 marks)**

|  |
| --- |
| Solution |
|   0.563  |
| Mathematical behaviours | Marks |
| * states probability inequality relevant to
* calculates one probability correctly
* calculates all probabilities correctly
 | 111 |

**Question 10(f) (1 mark)**

|  |
| --- |
| Solution |
|  as  |
| Mathematical behaviours | Marks |
| * obtains correct answer
 | 1 |

**Question 11(a)**  **(1 mark)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Mark |
| * evaluates
 | 1 |

**Question 11(b) (3 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * equates  and determines time taken to hit the ground
* differentiates to obtain
* calculates the speed
 | 111 |

**Question 11(c) (2 marks)**

|  |
| --- |
| Solution |
| Since  and  the ball is speeding up. |
| Mathematical behaviours | Marks |
| * differentiates  to determine  and states
* draws conclusion noting the same sign of both  and
 | 11 |

**Question 11(d) (1 mark)**

|  |
| --- |
| Solution |
|  Hence constant speed is attained. |
| Mathematical behaviours | Marks |
| * states  ie is constant
 | 1 |

**Question 11(e) (1 mark)**

|  |
| --- |
| Solution |
| A restriction on the domain is needed.ie   |
| Mathematical behaviours | Marks |
| * states restriction required on the domain
 | 1 |

**Question 12(a) (2 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * uses correct expression
* obtains correct answer
 | 11 |

**Question 12(b) (4 marks)**

|  |
| --- |
| Solution |
| If then and So and So and  |
| Mathematical behaviours | Marks |
| * expresses  in terms of
* expresses  in terms of
* calculates
* calculates
 | 1111 |

**Question 13(a) (5 marks)**

|  |
| --- |
| Solution |
|  Stationary Points: i.e.   or Now   =   Max tp At , At ,Now   equation of the function is   |
| Mathematical behaviours | Marks |
| * uses  to find stationary points
* substitutes into , and  to find which *x* value gives a local maximum turning point or clearly shows on sketch location of maximum and confirms maximum using 2nd derivative test
* integrates the derivative function correctly
* uses the point to determine the value of *c*
* states the correct equation of the function
 | 11111 |

**Question 13(b) (5 marks)**

|  |
| --- |
| Solution |
| (i) (ii)  |
| Mathematical behaviours | Mark |
| (i)* states correct volume to the nearest cubic centimetre

(ii)* states  in terms of
* uses incremental formula to obtain expression for small change in
* substitutes,  and
* states the decrease in capacity
 | 11111 |

**Question 14(a) (3 marks)**

|  |
| --- |
| Solution |
| Total number of cars in sample is Proportions of the various colours, and rounded to a whole multiple of :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| White | Black | Red | Blue | Other |
|  |  |  | 7 |  |

 |
| Mathematical behaviours | Marks |
| * obtains total sample size
* calculates all fractions correctly
* rounds all answers correctly
 | 111 |

**Question 14(b) (3 marks)**

|  |
| --- |
| Solution |
| Expected number of points per car So expected number of points per 100 cars  |
| Mathematical behaviours | Marks |
| * obtains correct expression for expected value
* calculates expected value (per car) correctly
* obtains correct answer
 | 111 |

**Question 14(c) (2 marks)**

|  |
| --- |
| Solution |
| Expected number of points per car (by colour)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| White | Black | Red | Blue | Other |
|  |  |  |  |  |

Since the expected points per car is greatest for Rodney’s red cars, Rodney is most likely to accumulate points fastest.  |
| Mathematical behaviours | Marks |
| * evaluates expected values correctly
* correct answer
 | 11 |

**Question 14(d) (2 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * uses correct formula
* evaluates correctly
 | 11 |

**Question 15(a) (4 marks)**

|  |
| --- |
| Solution |
| 1. none (consecutive selections are not independent so not binomial)
2. uniform
3. binomial
4. binomial
 |
| Mathematical behaviours | Marks |
| i)* states none

(ii) * states uniform

(iii)* states binomial

(iv)* states binomial
 | 1111 |

**Question 15(b) (4 marks)**

|  |
| --- |
| Solution |
| (i)  where  No, since  represents a probability and probability cannot be negative

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 4 | 6 | 8 | 10 |
|  | 0.05 | 0.30 | 0.25 | 0.4 |

(ii)  Yes as  and the sum of the probabilities is 1. |
| Mathematical behaviours | Marks |
| (i)* states no
* recognises negative probability

(ii)* states yes
* states both reasons
 | 1111 |

|  |
| --- |
| Solution |
|   or  as required |
| Mathematical behaviours | Marks |
| * anti-differentiates  to obtain  and uses  to get

 correct expression for .* anti-differentiates  to obtain and uses to get correct

 expression for * states required answer
 | 111 |

**Question 16(a) (3 marks)**

|  |
| --- |
| Solution |
| Distance travelled =  |
| Mathematical behaviours | Marks |
| * states the integral of the **absolute** velocity function from  to
 | 1 |

**Question 16(b) (1 mark)**

**Question 17(a) (2 marks)**

|  |
| --- |
| Solution |
| Define the random variable,  as the number of batteries that last for less than 2000 hours. Hence,    |
| Mathematical behaviours | Marks |
| * recognizes Binomial nature
* obtains correct answer
 | 11 |

**Question 17(b) (2 marks)**

|  |
| --- |
| Solution |
|   |
| Mathematical behaviours | Marks |
| * recognizes binomial nature
* obtains correct answer
 | 11 |

**Question 17(c) (2 marks)**

|  |
| --- |
| Solution |
| From part (b) we can conclude that there is an 85.6% chance that no more than 15 batteries out of 120 last less than 2000hrs. This would imply that there is only a 14.4% chance that more than 15 out of 120 batteries last less than 2000hrs.Hence the test does not imply compelling evidence that the manufacturer’s claim is false. |
| Mathematical behaviours | Marks |
| * obtains correct answer
* gives valid reason
 | 11 |

**Question 18(a) (3 marks)**

|  |
| --- |
| Solution |
|

|  |  |  |  |
| --- | --- | --- | --- |
| Outcome | Death | Permanent Disability | No payout |
| Profit |  |  |  |
| Probability |  |  |  |

 |
| Mathematical behaviours | Marks |
| * completes Probability row of table correctly
* completes exactly 2 entries of Profit row of table correctly
* completes table correctly
 | 111 |

**Question 18(b) (2 marks)**

|  |
| --- |
| Solution |
|  Hence the expected profit is $300 |
| Mathematical behaviours | Marks |
| * states correct formula for
* obtains correct answer
 | 11 |

**Question 18(c) (3 marks)**

|  |
| --- |
| Solution |
|  or  Note: CAS screen above shows   |
| Mathematical behaviours | Marks |
| * demonstrates calculations required to obtain variance
* obtains variance
* obtains standard deviation
 | 111 |

**Question 19(a) (1 mark)**

|  |
| --- |
| Solution |
| = |
| Mathematical behaviours | Marks |
| * states correct answer
 | 1 |

**Question 19(b) (1 mark)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * states correct answer
 | 1 |

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * states correct answer
 | 1 |

**Question 19(c) (1 mark)**

**Question 19(d) (2 marks)**

|  |
| --- |
| Solution |
| Shaded area marked M   |
| Marking key/mathematical behaviours | Marks |
| * recognises area of rectangle subtract
* states correct answer
 | 11 |

|  |
| --- |
| (i) Correct statement is (ii) Use CAS and solve for : Solve (  |
| Mathematical behaviours | Marks |
| (i)* chooses correct statement

(ii)* solves for *k*
 | 11 |

**Question 19(e) (2 marks)**

**Question 20(a) (3 marks)**

|  |
| --- |
| Solution |
| For the circle, For the square,  Hence,   |
| Mathematical behaviours | Mark |
| * demonstrates that  and states expression for the area of the circle
* demonstrates that side length = and states expression for the

 area of the square* concludes formula for
 | 111 |

**Question 20(b) (5 marks)**

|  |
| --- |
| Solution |
|  Or, to establish minimum has been achieved at ,  states coefficient of  is positive, hence minimum turning point or demonstrates with graph Hence the minimum total area is obtained when   |
| Mathematical behaviours | Marks |
| * determines
* equates and solves
* establishes hence a minimum
* determines  for  and  OR

 demonstrates through graph or coefficient of that  is a  quadratic with a minimum turning point  * concludes minimum area is when
 | 11111 |