**MATHEMATICS METHODS**

**MAWA Semester 1 (Unit 3) Examination 2019**

**Calculator-assumed**

# Marking Key

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**The release date for this exam and marking scheme is 14th June.**

**Section Two: Calculator-assumed (100 Marks)**

**Question 8(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)  (ii)  =  =  =  =  = | |
| Mathematical behaviours | Marks |
| * states * uses linearity and additivity to deduce * anti-differentiates * determines correct result of 16 | 1  1  1  1 |

**Question 8(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| =  =  =  = | |
| Mathematical behaviours | Marks |
| * anti-differentiates correctly * substitutes limits of integration correctly * determines exact result | 1  1  1 |

**Question 9(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Mark |
| * states  justifies answer using anti-differentiation | 1 |

**Question 9(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| For stationary points, | |
| Mathematical behaviours | Marks |
| * states  coordinates of stationary points | 1 |

**Question 9(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| and    Hence there is a change in concavity at and  so there is a horizontal point of inflection at . Hence *m* = 1. | |
| Mathematical behaviours | Marks |
| * states  and * demonstrates change in concavity at * states that horizontal point of inflection occurs at *m* = 1. | 1  1  1 |

**Question 9(d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * sketch shows  and roots * clearly shows  coordinate of minimum turning point * graphs correct shape and clearly labels points of inflection | 1  1  1 |

**Question 10(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| has a binomial distribution with parameters and  ie | |
| Mathematical behaviours | Marks |
| * identifies binomial distribution and states parameters | 1 |

**Question 10(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 10(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| (from calculator) | |
| Mathematical behaviours | Marks |
| * states a probability inequality relevant to one of the  values * calculates one probability correctly * calculates all probabilities correctly | 1  1  1 |

**Question 10(d) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| as | |
| Mathematical behaviours | Marks |
| * obtains correct answer | 1 |

**Question 10(e) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| 0.563 | |
| Mathematical behaviours | Marks |
| * states probability inequality relevant to * calculates one probability correctly * calculates all probabilities correctly | 1  1  1 |

**Question 10(f) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| as | |
| Mathematical behaviours | Marks |
| * obtains correct answer | 1 |

**Question 11(a)**  **(1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Mark |
| * evaluates | 1 |

**Question 11(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * equates  and determines time taken to hit the ground * differentiates to obtain * calculates the speed | 1  1  1 |

**Question 11(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since  and  the ball is speeding up. | |
| Mathematical behaviours | Marks |
| * differentiates  to determine  and states * draws conclusion noting the same sign of both  and | 1  1 |

**Question 11(d) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Hence constant speed is attained. | |
| Mathematical behaviours | Marks |
| * states  ie is constant | 1 |

**Question 11(e) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| A restriction on the domain is needed.  ie | |
| Mathematical behaviours | Marks |
| * states restriction required on the domain | 1 |

**Question 12(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * uses correct expression * obtains correct answer | 1  1 |

**Question 12(b) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| If then and  So and  So and | |
| Mathematical behaviours | Marks |
| * expresses  in terms of * expresses  in terms of * calculates * calculates | 1  1  1  1 |

**Question 13(a) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| Stationary Points:  i.e.  or  Now  =    Max tp  At , At ,    Now      equation of the function is | |
| Mathematical behaviours | Marks |
| * uses  to find stationary points * substitutes into , and  to find which *x* value gives a local maximum turning point or clearly shows on sketch location of maximum and confirms maximum using 2nd derivative test * integrates the derivative function correctly * uses the point to determine the value of *c* * states the correct equation of the function | 1  1  1  1  1 |

**Question 13(b) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)    (ii) | |
| Mathematical behaviours | Mark |
| (i)   * states correct volume to the nearest cubic centimetre   (ii)   * states  in terms of * uses incremental formula to obtain expression for small change in * substitutes,  and * states the decrease in capacity | 1  1  1  1  1 |

**Question 14(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Total number of cars in sample is  Proportions of the various colours, and rounded to a whole multiple of :   |  |  |  |  |  | | --- | --- | --- | --- | --- | | White | Black | Red | Blue | Other | |  |  |  | 7 |  | | |
| Mathematical behaviours | Marks |
| * obtains total sample size * calculates all fractions correctly * rounds all answers correctly | 1  1  1 |

**Question 14(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Expected number of points per car  So expected number of points per 100 cars | |
| Mathematical behaviours | Marks |
| * obtains correct expression for expected value * calculates expected value (per car) correctly * obtains correct answer | 1  1  1 |

**Question 14(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Expected number of points per car (by colour)   |  |  |  |  |  | | --- | --- | --- | --- | --- | | White | Black | Red | Blue | Other | |  |  |  |  |  |   Since the expected points per car is greatest for Rodney’s red cars, Rodney is most likely to accumulate points fastest. | |
| Mathematical behaviours | Marks |
| * evaluates expected values correctly * correct answer | 1  1 |

**Question 14(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * uses correct formula * evaluates correctly | 1  1 |

**Question 15(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| 1. none (consecutive selections are not independent so not binomial) 2. uniform 3. binomial 4. binomial | |
| Mathematical behaviours | Marks |
| i)   * states none   (ii)   * states uniform   (iii)   * states binomial   (iv)   * states binomial | 1  1  1  1 |

**Question 15(b) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| (i)  where  No, since  represents a probability and  probability cannot be negative   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | 4 | 6 | 8 | 10 | |  | 0.05 | 0.30 | 0.25 | 0.4 |   (ii)  Yes as  and the sum of the probabilities is 1. | |
| Mathematical behaviours | Marks |
| (i)   * states no * recognises negative probability   (ii)   * states yes * states both reasons | 1  1  1  1 |

|  |  |
| --- | --- |
| Solution | |
| or  as required | |
| Mathematical behaviours | Marks |
| * anti-differentiates  to obtain  and uses  to get   correct expression for .   * anti-differentiates  to obtain and uses to get correct   expression for   * states required answer | 1  1  1 |

**Question 16(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Distance travelled = | |
| Mathematical behaviours | Marks |
| * states the integral of the **absolute** velocity function from  to | 1 |

**Question 16(b) (1 mark)**

**Question 17(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Define the random variable,  as the number of batteries that last for less than 2000 hours. Hence, | |
| Mathematical behaviours | Marks |
| * recognizes Binomial nature * obtains correct answer | 1  1 |

**Question 17(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * recognizes binomial nature * obtains correct answer | 1  1 |

**Question 17(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| From part (b) we can conclude that there is an 85.6% chance that no more than 15 batteries out of 120 last less than 2000hrs. This would imply that there is only a 14.4% chance that more than 15 out of 120 batteries last less than 2000hrs.  Hence the test does not imply compelling evidence that the manufacturer’s claim is false. | |
| Mathematical behaviours | Marks |
| * obtains correct answer * gives valid reason | 1  1 |

**Question 18(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  |  | | --- | --- | --- | --- | | Outcome | Death | Permanent Disability | No payout | | Profit |  |  |  | | Probability |  |  |  | | |
| Mathematical behaviours | Marks |
| * completes Probability row of table correctly * completes exactly 2 entries of Profit row of table correctly * completes table correctly | 1  1  1 |

**Question 18(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence the expected profit is $300 | |
| Mathematical behaviours | Marks |
| * states correct formula for * obtains correct answer | 1  1 |

**Question 18(c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| or    Note: CAS screen above shows | |
| Mathematical behaviours | Marks |
| * demonstrates calculations required to obtain variance * obtains variance * obtains standard deviation | 1  1  1 |

**Question 19(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| = | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 19(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 19(c) (1 mark)**

**Question 19(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Shaded area marked M | |
| Marking key/mathematical behaviours | Marks |
| * recognises area of rectangle subtract * states correct answer | 1  1 |

|  |  |
| --- | --- |
| (i) Correct statement is  (ii) Use CAS and solve for : Solve ( | |
| Mathematical behaviours | Marks |
| (i)   * chooses correct statement   (ii)   * solves for *k* | 1  1 |

**Question 19(e) (2 marks)**

**Question 20(a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| For the circle, For the square,    Hence, | |
| Mathematical behaviours | Mark |
| * demonstrates that  and states expression for the area of the circle * demonstrates that side length = and states expression for the   area of the square   * concludes formula for | 1  1  1 |

**Question 20(b) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| Or, to establish minimum has been achieved at ,  states coefficient of  is positive, hence minimum turning point  or demonstrates with graph  Hence the minimum total area is obtained when | |
| Mathematical behaviours | Marks |
| * determines * equates and solves * establishes hence a minimum * determines  for  and  OR   demonstrates through graph or coefficient of that  is a  quadratic with a minimum turning point     * concludes minimum area is when | 1  1  1  1  1 |